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MATDIP301

Third Semester B.E. Degree Examination, July/August 2021

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: Answer any FIVE full questions.

1 a. Express the complex number $\frac{2+i}{3-4i}$ in $a + ib$ form. **(06 Marks)**

b. Express the complex number $1+\cos\alpha + i\sin\alpha$ in the modulus and argument form. **(07 Marks)**

c. Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^4(\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3(\cos 5\theta + i\sin 5\theta)^{-4}}$. **(07 Marks)**

2 a. Find the n^{th} derivative of $y = e^{ax}\cos(bx+c)$. **(06 Marks)**

b. If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$. **(07 Marks)**

c. Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots\ldots$ by using Maclaurin's expansion. **(07 Marks)**

3 a. In usual notations, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Prove that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ cuts orthogonally. **(07 Marks)**

c. Find the pedal equation for $r^m = a^m \cos m\theta$. **(07 Marks)**

4 a. Prove the Euler's theorem in the form $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = nU$. **(06 Marks)**

b. If $U = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, prove that:

$$\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 = \left(\frac{\partial U}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial U}{\partial \theta}\right)^2$$ **(07 Marks)**

c. If $U = x + y + z$, $V = y - z$, $W = z$ find the Jacobian $J = \frac{\partial(U,V,W)}{\partial(x,y,z)}$. **(07 Marks)**

5 a. Find the Reduction formula for $\int \sin^n x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^1\int_0^{y^2} xy\,dxdy$. **(07 Marks)**

c. Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \sin\theta\,dr\,d\theta\,d\phi$ **(07 Marks)**

6 a. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ **(06 Marks)**

b. Derive the relation between beta and gamma functions as $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \cdot \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

7 a. Solve $(x+y+1)^2 \frac{dy}{dx} = 1$ **(06 Marks)**

b. Solve $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$ **(07 Marks)**

c. Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ **(07 Marks)**

8 a. Solve $(D^3 - 3D^2 + 3D - 1)y = 0$ **(06 Marks)**

b. Solve $(D^2 - 5D + 6)y = 2e^{5x}$ **(07 Marks)**

c. Solve $(D^2 + D + 1)y = \sin 2x$ **(07 Marks)**

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